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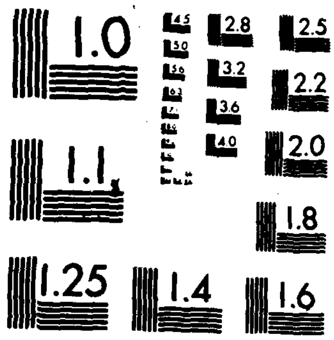

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Component Relevancy in Multistate Systems

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Abstract

~~We~~ The authors define a hierarchy of six successively weaker conditions for component relevancy in a multistate structure of M+1 performance levels. ~~We~~ They show that the six conditions are distinct except for M=1,2. ~~We present~~ Presented are basic structural properties corresponding to the six conditions: (a) the definition and properties of the dual structure, (b) redundancy at a lower level is preferable to redundancy at a higher level, and (c) the definition and properties of the structural importance of components.

0. Introduction and Summary.

In defining a binary coherent system, a key requirement is that each component be relevant to system functioning or failure. This requirement avoids the possibility of components that play no role in system functioning or failure. Only one natural definition of relevancy is possible.

In the multistate model, where each component may be in any of states $0, 1, \ldots, M > 1$, a variety of alternative relevancy requirements are possible. In this paper we list six reasonable relevancy requirements, forming a hierarchy of increasingly weaker requirements.

In Section 1, we present notation and terminology. Section 2 lists the six relevancy conditions ranging from the strongest to the weakest. We show that the six conditions are distinct except for the cases $M = 1,2$. Finally, in Section 3, we present basic structural properties corresponding to the six relevancy conditions. Thus, we define and study the dual structure, obtain the well known design principle that redundancy at a lower level is preferable to redundancy at a higher level, and define and study the structural importance of components.

1. Notation and Terminology.

The vector $\underline{x}$ $(x_1, \ldots, x_n)$ denotes the vector of states of components $1, \ldots, n$.

$C = \{1, \ldots, n\}$ denotes the set of component indices.

$(j_i, \underline{x}) \equiv (x_1, \ldots, x_{i-1}, j, x_{i+1}, \ldots, x_n)$, where $j = 0,1, \ldots, M$.

$(\cdot_i, \underline{x}) \equiv (x_1, \ldots, x_{i-1}, \cdot, x_{i+1} \ldots, x_n)$.

$\underline{j} \equiv (j, \ldots, j)$, where $j = 0,1, \ldots, M$.

$x \vee y \equiv \max(x,y)$.

$\underline{x} \vee \underline{y} \equiv (x_1 \vee y_1, \ldots, x_n \vee y_n)$.

$x \wedge y \equiv \min(x,y)$.

$x \wedge y \equiv (x_1 \wedge y_1, \ldots, x_n \wedge y_n)$.

When we say $\phi(x_1, \ldots, x_n)$ is nondecreasing we mean ϕ is nondecreasing in each argument.

Given a set S, S^n denotes its $n^{\underline{th}}$ Cartesian power.

2. Levels of Component Relevancy.

A basic ingredient in the theory of binary coherent systems is the structure function ϕ: $\{0,1\}^n \to \{0,1\}$ which determines the state of the system in terms of the states of the n components. The following two conditions are required for a binary system to be a coherent structure [1, Def. 2.1, p.6]:

(i) The function $\phi(\underline{x})$ is nondecreasing.

(ii) For each i there exists a vector $(\cdot_i, \underline{x})$ such that $\phi(1_i, \underline{x}) > \phi(0_i, \underline{x})$.

Condition (i) expresses the reasonable assumption that improving component performance should not degrade system performance. Condition (ii) asserts that each component is relevant to system performance, thus eliminating from consideration components that have no effect on system performance. It follows from (i) and (ii) that

(iii) $\phi(\underline{1}) = 1$ and $\phi(\underline{0}) = 0$.

Recently researchers felt the need to develop the theory of multistate coherent systems to describe more adequately the performance of components and systems which have more than two levels of performance. Again a basic

ingredient in such a theory is the structure function ϕ: $S^n \to S$, where $S = \{0,1, \ldots, M\}$ is the set representing levels of performance varying from perfect functioning M to total failure 0. (We concentrate in this paper on the case where S is finite.) One possible approach to extend the concept of binary coherent structures is to impose on ϕ a set of "reasonable" conditions which generalize conditions (i) and (ii). Condition (i) is extended in a straightforward manner by requiring ϕ to be nondecreasing. It turns out, however, that the relevancy condition can be extended in many different ways, each leading to a distinct class of multistate "coherent" structures. In a multistate model, relevancy becomes a more complex concept that admits different mathematical formulations. The following are successively weaker versions of relevancy:

(ii)′ For every component i, there exists a vector $(\cdot_i,\underline{x})$ such that $\phi(j_i,\underline{x}) = j$, $j = 0,1, \ldots, M$.

(ii)″ For every component i and level j, there exists a vector $(\cdot_i,\underline{x})$ such that $\phi(j_i,\underline{x}) = j$ while $\phi(\ell_i,\underline{x}) \neq j$ for $\ell \neq j$.

(ii)‴ For every component i and level $j \geq 1$, there exists a vector $(\cdot_i,\underline{x})$ such that $\phi(j_i,\underline{x}) \geq j$ and $\phi((j-1)_i,\underline{x}) \leq j-1$.

(ii)$^{(iv)}$ For every component i and level $j \geq 1$, there exists a vector $(\cdot_i,\underline{x})$ such that $\phi((j-1)_i,\underline{x}) < \phi(j_i,\underline{x})$.

(ii)$^{(v)}$ For every component i and level $j \geq 1$, there exists a vector $(\cdot_i,\underline{x})$ such that $\phi(0_i,\underline{x}) < \phi(j_i,\underline{x})$.

(ii)$^{(vi)}$ For every component i, there exists a vector $(\cdot_i,\underline{x})$ such that $\phi(0_i,\underline{x}) < \phi(M_i,\underline{x})$.

Among the six conditions above, the first five indicate a degree of relevancy of each component to every level of performance, while the last merely states that every component is relevant to the system. Condition

$(ii)''$ is due to El-Neweihi, Proschan, and Sethuraman (1978), Condition $(ii)'''$ is due to Natvig (1982), Conditions $(ii)^{(iv)}$, $(ii)^{(v)}$ are due to Griffith (1980), and the remaining conditions are new. It should also be remarked that the condition

$$(iii)' \quad \phi(\underline{j}) = j, \ j = 0,1, \ldots, M,$$

which generalizes Condition (iii) is not necessarily satisfied by a non-decreasing structure function ϕ which satisfies one of the relevancy axioms (except, of course, when $M = 1$).

When $M = 1$ all the above relevancy conditions are equivalent; also when $M = 2$ Conditions $(ii)'$ and $(ii)''$ are equivalent. The following examples show that in general the above relevancy axioms are not equivalent.

<u>Example 2.1.</u> Let $n = 2, M = 3$. Define ϕ by $\phi(0,0) = 0$, $\phi(1,0) = \phi(0,1) = \phi(1,1) = 1$, $\phi(0,2) = \phi(0,3) = \phi(1,2) = \phi(2,0) = \phi(2,1) = \phi(2,2) = \phi(3,0) = 2$, $\phi(1,3) = \phi(2,3) = \phi(3,1) = \phi(3,2) = \phi(3,3) = 3$. Then ϕ satisfies Condition $(ii)''$ but ϕ does not satisfy Condition $(ii)'$.

<u>Example 2.2.</u> Let $n = 2, M = 2$. Define ϕ by $\phi(0,0) = 0$, $\phi(0,1) = \phi(0,2) = \phi(1,0) = \phi(1,1) = \phi(1,2) = \phi(2,0) = \phi(2,1) = 1$, $\phi(2,2) = 2$. Then ϕ satisfies Condition $(ii)'''$ but ϕ does not satisfy Condition $(ii)''$.

<u>Example 2.3.</u> Let $n = 2, M = 2$. Define ϕ by $\phi(0,0) = \phi(1,0) = \phi(0,1) = 0$, $\phi(1,1) = (2,1) = (2,0) = (0,2) = 1$, $(1,2) = (2,2) = 2$. Then ϕ satisfies Condition $(ii)^{(iv)}$ but does not satisfy Condition $(ii)'''$.

<u>Example 2.4.</u> Let $n = 2, M = 2$. Define ϕ by $\phi(0,0) = 0$, $\phi(0,1) = \phi(1,0) = \phi(1,1) = \phi(2,0) = \phi(2,1) = 1$, $\phi(0,2) = \phi(1,2) = \phi(2,2) = 2$. Then ϕ satisfies Condition $(ii)^{(v)}$ but ϕ does not satisfy Condition $(ii)^{(iv)}$.

Example 2.5. Let $n = 2$, $M = 2$. Define ϕ by $\phi(0,0) = \phi(1,0) = 0$, $\phi(0,1) = \phi(0,2) = \phi(1,1) = \phi(1,2) = \phi(2,0) = \phi(2,1) = 1$, $\phi(2,2) = 2$. Then ϕ satisfies Condition $(ii)^{(vi)}$ but does not satisfy Condition $(ii)^{(v)}$.

A class of nondecreasing structure functions which satisfy (iii)′ and one of the relevancy axioms may be designated as a class of multistate coherent systems. In the following section we examine some interesting properties and concepts for such classes which are closely related to the relevancy axioms.

3. Structural Properties Related To Relevancy.

As in the binary case, a dual structure for each multistate structure can be defined:

Definition 3.1. Let ϕ be the structure function of a multistate system. The dual structure function ϕ^D is given by:

$$\phi^D(\underline{x}) = M - \phi(\underline{M} - \underline{x}).$$

The following interesting question naturally arises: Do the components of the dual structure ϕ^D inherit the relevancy property enjoyed by the components of ϕ? The following theorem asserts that with the exception of Condition $(ii)^{(v)}$, the answer is yes.

Theorem 3.2. Let ϕ be a multistate structure function which satisfies one of the Conditions (ii)′ through $(ii)^{(iv)}$ or Condition $(ii)^{(vi)}$. Then the dual structure function ϕ^D satisfies the same condition.

Proof. The proof is straightforward and is therefore omitted.

Example 3.3. Let ϕ be defined as in Example 2.4. Then ϕ satisfies Condition $(ii)^{(v)}$ but ϕ^D does not satisfy the same condition.

Design engineers have used the well known principle that redundancy at the component level is preferable to redundancy at the system level. This principle still holds in the multistate model and is translated into mathematical form in (a) of the following theorem; (b) is a dual result.

<u>Theorem 3.4.</u> Let ϕ be a nondecreasing multistate structure function. Then

(a) $\phi(\underline{x} \vee \underline{y}) \geq \phi(\underline{x}) \vee \phi(\underline{y})$ for all $\underline{x}$ and $\underline{y}$.

(b) $\phi(\underline{x} \wedge \underline{y}) \leq \phi(\underline{x}) \wedge \phi(\underline{y})$ for all $\underline{x}$ and $\underline{y}$.

The above theorem is an immediate consequence of the monotonicity of ϕ. The following less trivial result is due to El-Neweihi, Proschan, and Sethuraman (1978):

Let the nondecreasing structure function ϕ also satisfy Condition (ii)$''$. Then equality holds in (a) ((b)) for all $\underline{x}$ and $\underline{y}$ implies that the system is parallel (series).

An extension to the above result is achieved by Griffith (1980) by replacing (ii)$''$ with the weaker Condition (ii)$^{(iv)}$. However, Griffith (1980) showed by an example that the same result is not true if (ii)$''$ is replaced by (ii)$^{(vi)}$. The following example shows that Condition (ii)$^{(v)}$ is not sufficiently strong.

<u>Example 3.5.</u> Let $n = 2$, $M = 2$. Let ϕ be as defined in Example 2.4. Then ϕ satisfies Condition (ii)$^{(v)}$, $\phi(\underline{x} \vee \underline{y}) = \phi(\underline{x}) \vee \phi(\underline{y})$ for all $\underline{x}$ and $\underline{y}$ but $\phi(\underline{x}) \neq \max\limits_{1\leq i\leq n} x_i$

A measure of the structural importance of each component to a given system is of obvious practical significance. In the binary case the importance of component i to a coherent structure ϕ is given by

$I(i) = \frac{1}{2^{n-1}}$ Card$\{(\cdot_i,\underline{x}): \ \phi(0_i,\underline{x}) < \phi(1_i,\underline{x})\}$. Note that the relevancy Condition (ii) guarantees that $I(i) > 0$ for each $i = 1, \ldots, n$. Generalizations of such a measure in the multistate setting are now evident. Let ϕ be a nondecreasing multistate structure. A measure of the importance of component i to the structure ϕ can be given by

$I'(i) = \frac{1}{(M+1)^{n-1}}$ Card$\{(\cdot_i,\underline{x}): \ \phi(j_i,\underline{x}) = j, \ j = 0,1, \ldots, M\}$. Note t $I'(i) > 0$ for each i if and only if ϕ satisfies Condition (ii)$'$. ' measures $I''(i)$, $I'''(i)$, $I^{(iv)}(i)$, $I^{(v)}(i)$, and $I^{(vi)}(i)$ are similar.y defined. Note that $I'(i) \le I''(i) \le I'''(i) \le I^{(iv)}(i) \le I^{(v)}(i) \le I^{(vi)}(i)$. Also observe that each of those six measures of structural importance has its natural probabilistic counterpart (see Block and Savits (1982)).

Finally, we shed some light on preservation of the various relevancy axioms under modular decomposition. A question raised and answered by Griffith (1980) is whether a "relevant" component within a "relevant" module is "relevant" in the system. The answer is yes if relevancy is defined in terms of Conditions (ii)$''$ and (ii)$^{(iv)}$. However, an example is given by Griffith (1980) to show that this is not necessarily the case for Condition (ii)$^{(vi)}$. It can be easily shown that the answer is still yes if relevancy is defined in terms of Conditions (ii)$'$ and (ii)$'''$. The following example shows that this is not necessarily the case for Condition (ii)$^{(v)}$.

<u>Example 3.6.</u> Let $n = 2$, $M = 2$. Define ϕ by $\phi(0,0) = \phi(1,0) = \phi(2,0) = 0$, $\phi(0,1) = \phi(1,1) = \phi(2,1) = \phi(0,2) = 1$, $\phi(1,2) = \phi(2,2) = 2$. Now let $\psi(x_1,x_2,x_3) = \phi(\phi(x_1,x_2),x_3)$.

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